

NAGI-1483  
FINAL  
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46145**Preliminary Design Methodology for Deployable Spacecraft Beams****Martin M. Mikulas, Jr. and Bryce L. Cox****ABSTRACT**

There is currently considerable interest in low-cost, lightweight, compactly packagable deployable elements for various future missions involving small spacecraft. These elements must also have a simple and reliable deployment scheme and possess zero or very small free-play. Although most small spacecraft do not experience large disturbances, very low stiffness appendages or free-play can couple with even small disturbances and lead to unacceptably large attitude errors which may involve the introduction of a flexible-body control system. A class of structures referred to as "rigidized structures" offer significant promise in providing deployable elements that will meet these needs for small spacecraft. The purpose of this paper is to introduce several new rigidizable concepts and to develop a design methodology which permits a rational comparison of these elements to be made with other concepts.

**INTRODUCTION**

There is currently considerable interest in low-cost, lightweight, compactly packagable deployable structural elements for various future space missions involving small spacecraft<sup>2</sup>. In addition to these requirements, simplicity and reliability of deployment are of paramount concern. In many instances the concern over the cost and reliability of deployable components leads spacecraft designers to either not consider them at all, or to use existing deployable components with low stiffness or joint deadbands. In either case, spacecraft performance for such missions can be severely compromised due to the lack of well accepted, high performance deployable components.

Although most small spacecraft do not experience large disturbances, very low stiffness appendages can couple with even small disturbances and lead to unacceptably large attitude errors which may involve the introduction of a flexible modes control capability onboard which increases spacecraft cost. Thus, there exists a need in small spacecraft for stiff deployable components which are truly low cost and reliable.

The current commercially available SOA for deployable beam elements includes the unfurlable STEM, the continuous-coilable-longeron mast, the FASTMAST as used for the Tether satellite, and unfolding "Lazy-Tong" devices which deploy a few bays of panels such as on the SEASAT. The other approach used in deployment is to simply hinge panels or elements together with no supporting structure. None of these available deployable devices satisfies all of the desired requirements for the new generation of lighter, faster, and cheaper, small spacecraft.

A class of structures referred to as "rigidized structures"<sup>3</sup> offer significant promise in providing high performance structural components for the new small spacecraft. A large reflector based on inflatable and rigidized concepts is currently being built for an In-Step flight experiment<sup>4</sup> as a proof of concept for a microwave and VLBI antennas. Such reflectors have also been studied for optical interferometers<sup>5</sup> and solar concentrators<sup>6</sup>. The main difference between this paper and reference 1 is that the mass of the beam is included in all of the applicable equations. This provides a more accurate assessment of the performance potential of the new "rigidizable" structural concepts when compared to the alternate concepts.

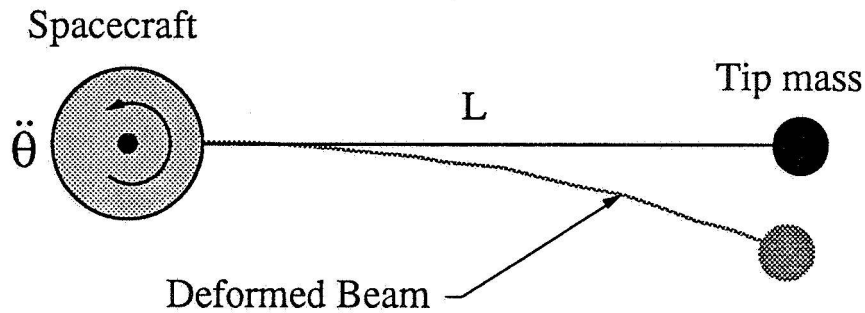
## BEAM DESIGN METHODOLOGY

### General Approach for Developing Beam Weight Equations

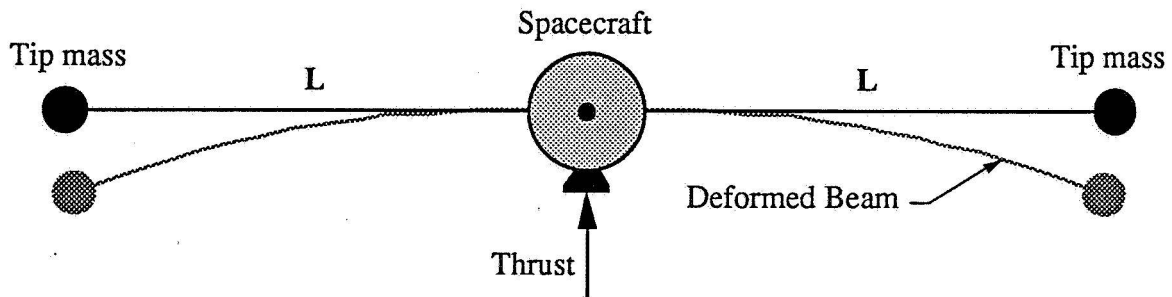
The primary purpose of this paper is to develop an approach to enable a rational comparison to be made of weight and diameter of different deployable beam concepts for small spacecraft. The four general beam concepts to be compared are as follows:

- a. Unfurlable STEM<sup>7</sup>
- b. Coilable longeron<sup>8</sup>
- c. Space rigidizable organic matrix composite (inflatably deployed)<sup>9</sup>
- d. Unfurlable thin walled aluminum (inflatably deployed)<sup>3,10,11</sup>

For purposes of comparing the relative merits of the different beam concepts, one of two possible moment inducing load conditions is applied to the deployable beam cantilevered from the spacecraft. One possible loading configuration is to have the beam supporting a tip mass cantilevered from a spacecraft with a root acceleration of  $\ddot{\theta}$  (Sketch a). The second loading condition is with the beam supporting the tip mass and cantilevered from a spacecraft that has a lateral acceleration loading from a fired thruster (Sketch b).



Sketch a.- Schematic of Spacecraft with Deployable Beam



Sketch b.- Schematic of Spacecraft with Two Deployable Beams

For the purposes of this paper, only the second case with the lateral acceleration loading will be considered for the numerical examples. However, it should be pointed out that for any lateral acceleration loading there is a comparable root acceleration. This will be demonstrated in a later section.

Although there are numerous factors which contribute to the concept selection and design of deployable beams for spacecraft, attention in this paper is focused on two primary design drivers. These are 1) the moment constraint resulting from the lateral acceleration loading shown in Sketch b, and 2) a lowest beam natural frequency constraint.

The constraint on frequency is commonly imposed upon spacecraft components to deal with flexible control issues, while the constraint associated with a moment imposes a loading that the beam must be able to withstand without failure. A general description of the four beams considered in this paper and how they are modeled are given in the next sections.

**a. Unfurlable STEM.-** This class of structures involves materials which are thin enough to be rolled up for packaging without yielding, and subsequently unfurled into the deployed state. The classic example of this type of structure is the STEM and BISTEM<sup>7</sup>. The STEM structure is a thin metallic sheet which is coiled into a compact cylindrical roll and deployed on-orbit. The design is such that no involved bending strains exceed yield. These structural elements unfurl into a long tubular shape, thus forming a deployed beam. The major shortcomings of these elements is that a slit is required along the length to accommodate low strain packaging, resulting in very low beam torsional stiffness, and that the deployment mechanism is quite heavy. The BISTEM beam is composed of two interwoven STEMS which provides additional torsional stiffness from the resulting friction between the overlapping elements. Under some loading conditions a slippage can occur between the overlapping elements resulting in unwanted deformation or dynamic perturbations. In the present paper the STEM is treated as a simple steel tube with a wall thickness of 0.005" (5 mils).

**b. Coilable longeron.-** The coilable longeron beam is a highly used deployable beam and is well described in reference 8. The weight and performance equations for this beam are taken from ref. 8. The popularity of this beam arises from its high reliability and wide experience base. Its shortcomings are that it requires a relatively heavy canister and is limited in size to about 20" in diameter due to high straining in the stowed condition.

**c. Space rigidizable organic matrix composite (inflatably deployed).-** This concept is basically a simple tubular beam fabricated from a fiber fabric impregnated with a matrix that is rigidized<sup>9</sup> after pressure deployment in space. This concept is still in the development stage, however, it offers the promise of a very simple, low-cost, compactly packagable beam. In the present paper the tube is considered to be fabricated from a bi-directional KEVLAR fabric impregnated with a rigidizable matrix. The effective properties assumed for the beam material are:  $E = 4 \times 10^6$  psi, thickness = 0.011", and a weight density of 0.05 lb/in<sup>3</sup>.

**d. Unfurlable thin-walled aluminum (inflatably deployed).-** This concept is basically a thin-walled aluminum tubular beam pressure deployed in space. For this approach the tubes are made from thin (~3 mil) low-yield-stress aluminum sandwiched between two thin layers of reinforced Kapton film for structural strength and initial inflatant

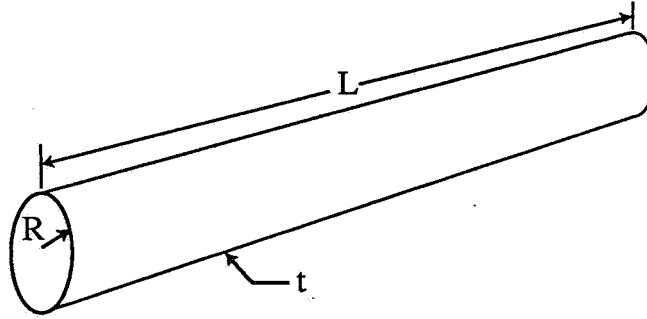
containment<sup>3,10</sup>. The deployment is obtained by pressurization of the tube to a cylindrical shape. After deployment the pressure is increased to yield the thin aluminum into its final wrinkle-free state. The tube is then de-pressurized and remains in a cylindrical shape providing a high performance structural member. Although this concept is still in the development stage a full scale deployable solar array has been built and ground demonstrated<sup>10</sup>. This concept has the potential for being an extremely simple, low-cost, and reliable deployment system. The primary shortcoming of this concept is the low level of development that has occurred in exploring different hybrid wall concepts. In the present paper the beam is simply considered to be fabricated from 0.003" thick aluminum with a modulus of  $10 \times 10^6$  psi.

### Weight of a Thin Walled Tubular Beam Subjected to Frequency and Root Moment Constraints

The weight of a thin walled tubular beam as shown in Sketch c can be written as:

$$W_{T.B.} = \rho AL = \rho(2\pi Rt)L \quad (T1)$$

where  $\rho$  is the weight density of the beam material, and  $R$  and  $t$  are radius and thickness of the tubular beam respectively.



Sketch c.- Schematic of tubular beam

**Frequency Constraint.-** For a cantilever beam with a tip mass,  $m_{tip}$ , and mass of the beam,  $m_{beam}$ , the first natural bending frequency  $f$  is approximated by beam theory as:

$$f = \frac{1}{2\pi} \sqrt{\frac{3EI}{L^3(m_{tip} + 0.227m_{beam})}} \quad (T2)$$

where  $m_{beam} = \frac{\rho(2\pi Rt)L}{g}$

and  $E$  is the extensional modulus of the tube material. If the tube is made of an orthotropic material,  $E$  is the extensional modulus in the long direction of the beam. The moment of inertia of a thin walled tubular beam is approximated by:

$$I = \pi R^3 t \quad (T3)$$

Substituting for I from equation T3 into equation T2 yields the following equation governing R and t:

$$R^3 t = \frac{L \bar{f}}{3E\pi} \quad (T4)$$

$$\text{where } \bar{f} = L^2 (m_{\text{tip}} + 0.227 m_{\text{beam}}) (2\pi f)^2$$

**Beam Root Moment Constraint.**- The root moment for a cantilevered beam with tip mass and beam mass subjected to a lateral acceleration loading, (s.g.)g is:

$$M = (f.s.)(s.g.)g\alpha \left( m_{\text{tip}}L + m_{\text{beam}} \frac{L}{2} \right) \quad (T5)$$

where  $\alpha$  is the dynamic overshoot factor due to a suddenly applied loading, f.s. is the factor of safety, and s.g. is the fraction of gravitational acceleration.  $\alpha$  and f.s. are coefficients like s.g., and will vary depending upon the problem considered.

The root stress  $\sigma$  in the cylinder due to this moment is:

$$\sigma = \frac{MR}{I} = \frac{MR}{\pi R^3 t} \quad (T6)$$

The failure mode in the tubular beam is assumed to be local wall buckling of the cylinder which is given by:

$$\sigma_{\text{local}} = C \frac{Et}{R} \quad (T7)$$

This is a generalization of the wall buckling equation for an isotropic cylinder which has a theoretical value of  $C = 0.6$ . In the present study, the constant C was determined for the particular orthotropic wall construction being considered.

Combining equations T5, T6, and T7, a second equation relating R and t is obtained as:

$$Rt^2 = \frac{(f.s.)(s.g.)g\alpha \left( m_{\text{tip}}L + m_{\text{beam}} \frac{L}{2} \right)}{\pi CE} \quad (T8)$$

**Solving the Constraint Equations for the Radii.**- In reference 1, a single closed form equation governing the weight of a tubular beam was found by solving equations T4 and T8 simultaneously and substituting the results for t and R into the weight equation T1. Now, however, because  $\bar{f}$  and M are both in terms of  $m_{\text{beam}}$ , which is a function of R and t, solving the equations simultaneously and obtaining a closed form solution becomes difficult. The equations, which are expanded forms of T4 and T8 become:

$$f = \frac{1}{2\pi} \sqrt{\frac{3E\pi R^3 t}{L^3 \left( m_{tip} + 0.227 \frac{\rho(2\pi R t)L}{g} \right)}} \quad (S1)$$

$$R = \frac{(f.s.)(s.g.)g\alpha \left( L m_{tip} + \frac{L^2 \rho(2\pi R t)}{2g} \right)}{\pi C E t^2} \quad (S2)$$

It can be seen that the addition of the beam mass complicates the expressions. Therefore, the practical constraints on the thicknesses, due to the compact packaging constraints on the different tubular beam concepts, must be utilized. This dictates values for the thicknesses of the respective beams, and now the value of R can be determined in terms of the frequency, or in terms of the moment. This is dealt with in the next section.

**Beam Weight Considering Thickness Constraints.** - For each of the tubular beam concepts considered in the present paper there is a limitation on the tube wall thickness. This limitation is imposed in order to compactly package the material without excessive damage. To account for this thickness constraint, weight equations are evaluated for both frequency and root moment constraints considering the thickness to be a constant.

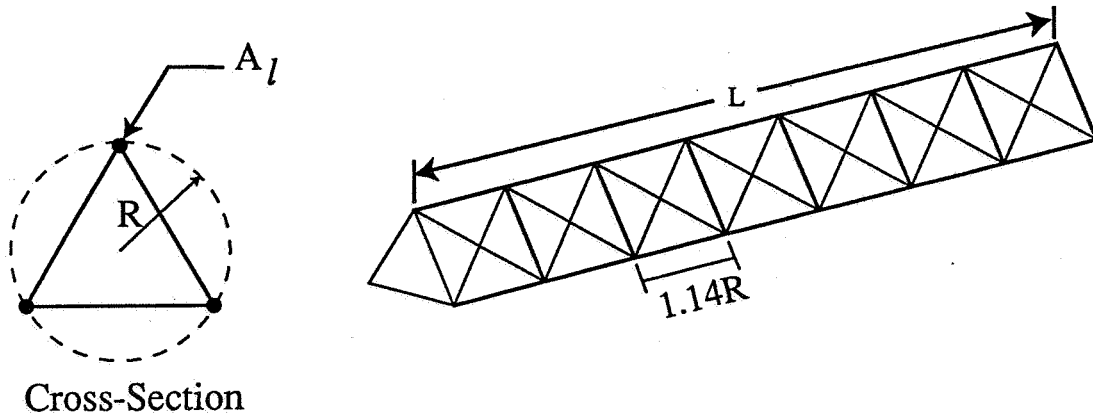
Due to the complexity of equations S1 and S2, imposed by the addition of the beam mass, *Mathematica* is relied upon to solve the constraint equations for their corresponding radii. Each radius, along with the thickness value for the tubular beam being considered, is then substituted into the weight equation T1. The result is two weight equations, the maximum of which is required to guarantee that both design constraints are met. (The complete *Mathematica* code can be found in Appendix A.) It should be noted that the weight corresponding to the frequency constraint increases for increasing thickness t while the weight corresponding to the root moment constraint decreases for increasing thickness t.

#### Weight of a Coilable Longeron Beam Subjected to Frequency and Root Moment Constraints

The weight  $W_{C.L.}$  of a coilable longeron beam as shown in Sketch d, is taken from reference 8 as:

$$W_{C.L.} = 3.4(3pA_l L) \quad (C1)$$

where  $A_l$  is the area of each longeron which is positioned at a radius R from the beam's centroid. For the coilable longeron beam, the two unknowns to be determined from the frequency and root moment constraints are  $A_l$  and R.



Sketch d. Schematic of coilable longeron beam

The quantities inside the parentheses of equation C1 are the weight of the three longerons and the factor 3.4 is an empirical constant which accounts for the beam's battens, diagonals and joints. This empirical constant is taken from reference 8 where it was determined by curve fitting data from several coilable longerons beams which had been built.

**Frequency Constraint.-** The frequency equation for a cantilevered coilable longeron beam with beam mass and a tip mass, is taken to be the same as that for the tubular beam and is given by equation T2. The bending moment of inertia of a three longeron beam about an axis passing through its centroid is given by:

$$I = 1.5A_l R^2 \quad (C2)$$

It should be noted that the moment of inertia of a three longeron beam is independent of the angle of the axis which passes through the beam's centroid. In other words the beam behaves elastically similar to a cylindrical beam with the same radius  $R$ , and the same amount of material at that radius.

Substituting the expression for  $I$  from equation C2 into the frequency equation T2, results in an equation governing  $R$  and  $A_l$  as:

$$R^2 A_l = \frac{L \bar{f}}{3(1.5)E} \quad (C3)$$

$$\text{where } m_{\text{beam}} = \frac{3.4(3pA_l L)}{g}$$

**Root Moment Constraint.-** The root moment is taken to be the same as that for the tubular beam and is given by equation T5 with the appropriate change to  $m_{\text{beam}}$ . For the coilable longeron beam, failure is assumed to be buckling of the root longeron due to compression from the root moment. Longerons buckling is taken as the simple support Euler load as:

$$P_{\text{Euler}} = \frac{\pi^2 E I_l}{(1.14R)^2} \quad (C4)$$

where the beam bay length is given in reference 8 as  $1.14R$ , and the moment of inertia  $I_\ell$  of a longeron is:

$$I_\ell = \frac{A_\ell^2}{4\pi} \quad (C5)$$

where 
$$A_\ell = \frac{\pi d^2}{4}$$

Combining equations C4, C5, T5, and T6 yields a second equation governing  $R$  and  $A_\ell$  as:

$$\frac{R}{A_\ell^2} = \frac{1.5\pi E}{(f.s.)(s.g.)g\alpha 4(1.14)^2 \left( m_{\text{tip}}L + m_{\text{beam}} \frac{L}{2} \right)} \quad (C6)$$

Equation C6 represents a second equation which governs the coilable longeron radius  $R$  and the longeron area  $A_\ell$ . In addition to equations C3 and C6, an additional constraint<sup>8</sup> must be imposed upon the longeron diameter to accommodate packaging as discussed in the next section.

**Longeron Packaging Constraint.**- For elastic packaging<sup>8</sup>, the longeron diameter,  $d$ , must be limited as follows:

$$\frac{d}{2R} = \epsilon = \frac{\sqrt{\frac{4A_\ell}{\pi}}}{2R} \quad (C7)$$

where  $\epsilon$  is the longeron allowable strain. In reference 8 this strain value was taken as 0.0133 for fiberglass and is the same value used in the present paper. It should also be pointed out there is a factor of 2 error in equation 12 of reference 8. The left hand side of equation 12 should read  $d/2R$  as in equation C7 rather than  $d/R$ .

**Coilable Longerons Beam Weight.** - Equations C3, C6, and C7 represent three equations for the two unknowns  $R$  and  $A_\ell$ , thus the design is over specified and must be separated into three possible design cases to determine which two of the three conditions govern. The three possible cases are: (1) impose the frequency constraint equation C3 and the root moment constraint equation C6, (2) impose the root moment constraint equation C6 and the stowage constraint C7, and (3) impose the frequency constraint equation C3 and the stowage constraint equation C7. Case 3 is never critical, thus, the higher of the two weights resulting from cases (1) and (2) must be taken as the coilable longeron weight.

As with the tubular beam, the addition of the beam mass to equations T2 and T5 complicate the constraint equations, C3 and C6 in this case. The expanded forms of C3 and C6 are:



$$f = \frac{1}{2\pi} \sqrt{\frac{3E(1.5)A_\ell R^2}{L^3 \left( m_{\text{tip}} + 0.227 \frac{3.4(3\rho A_\ell L)}{g} \right)}} \quad (\text{S3})$$

$$R = \frac{1.5\pi E A_\ell^2}{(\text{f.s.})(\text{s.g.})g\alpha 4(1.14)^2 \left( L m_{\text{tip}} + \frac{L^2 3.4(3\rho A_\ell)}{2g} \right)} \quad (\text{S4})$$

*Mathematica* is again used to solve the frequency constrained equation, S3, and the root moment constrained equation, S4, for their corresponding values of  $A_\ell$ . Each value of  $A_\ell$  is then substituted into the weight equation C1. The result is two weight equations, the maximum of which is required to guarantee that both design constraints are met. Similarly, *Mathematica* is used to solve the constraint equations for their corresponding radii, which are then used to obtain the plots of the coilable longeron diameters.

### EXAMPLE BEAM WEIGHT RESULTS AND DISCUSSION

A general comparison of the various deployable beam concepts constructed of different materials is difficult to make over a wide range of the frequency and loading design parameters  $\bar{f}$  and  $M$ . The reason for this is that the different beams are governed by practical constraints such as limitations on material thickness which in turn are a function of the level of the design parameters. In order to obtain insight into the relative weight and stowage efficiency of the deployable beam concepts considered in the present paper, a specific set of design requirements are selected which are considered to be representative of a range of typical spacecraft conditions. The following assumptions are made:

lateral acceleration loading (s.g.)g	= 0.015 g
natural bending frequency	= 0.02 - 1 Hz
beam length	= 3.5, 7, 14, 28 meters
tip mass (respective to length)	= 0.01, 0.04, 0.16, 0.16 lb·sec <sup>2</sup> /in
dynamic overshoot factor ( $\alpha$ )	= 1
factor of safety (f.s.)	= 1
local wall buckling constant (C)	= 0.2

The first requirement considered is a root moment constraint imposed by a 0.015 g lateral acceleration loading. As mentioned previously, the lateral acceleration loading has a corresponding root acceleration which can be found in the following manner using the moment due to an angular acceleration, and the moment due to a lateral acceleration loading as given by equation T5:

$$(m_{tip}L + m_{beam}\frac{L}{2})(s.g.)g = \left(L^2m_{tip} + \frac{L^2}{3}m_{beam}\right)\ddot{\theta} \quad (R1)$$

Solving for  $\ddot{\theta}$  yields:

$$\ddot{\theta} = \frac{\left(m_{tip} + \frac{m_{beam}}{2}\right)(s.g.)g}{\left(m_{tip} + \frac{m_{beam}}{3}\right)L} \quad (R2)$$

By adding the beam mass to equation R1, the angular acceleration solved for in R2 becomes beam dependent, and, therefore, material dependent. Also, since  $m_{beam}$  is a function of R for the tubular beams, the angular acceleration becomes dependent upon frequency, when the frequency constraint is used to find R. In the case of the coilable longeron,  $m_{beam}$  is a function of  $A_\ell$ , which is a function of frequency when the frequency/root moment constraint is used to find  $A_\ell$ . This change to angular acceleration, however, has no significant effect on the beam weights compared to the results when angular acceleration does not include the beam mass.

The other design requirement considered is that of a lowest natural bending frequency constraint. Since the value of this constraint is typically not well defined, it is varied in this study to determine its impact on structural weight and beam diameter.

The other spacecraft input needed to make a design study is the tip mass on the beam. To obtain representative mass values for small spacecraft, the inflatable solar array of ref. 10 was used as an example. This solar array was about 11.5' (3.5) meters long and weighed about 7 pounds. Since there are two beams that support this weight, half of the weight is assigned to each beam or  $m_{tip} = 3.5\text{lb}/386\text{in}/\text{sec}^2 \sim .01 \text{ lb} \cdot \text{sec}^2/\text{in}$ . For the current design study this mass is assumed to vary as a function of the square of the beam length to simulate area masses such as that associated with solar arrays.

To obtain insight into the relative weights and diameters of the deployable beams considered in the present paper, four beam lengths (3.5, 7, 14, and 28 meters) were investigated. For each length the beam weights and beam diameters are plotted as a function of natural frequency. The natural frequency was varied from .02 to 1 Hz to cover the range of interest for most spacecraft. The properties used for each of the four beams are presented in the following table.

	E, psi	$\rho$ , lb/in <sup>3</sup>	t, in
Aluminum	$10 \times 10^6$	.1	.003
Rigidizable	$4 \times 10^6$	.05	.011
Steel STEM	$30 \times 10^6$	.3	.005
Coilable Longerons	$7.5 \times 10^6$	.07	-----

Since the purpose of this paper is to present the methodology of preliminary design, the values of  $\alpha$  and f.s. are both set equal to 1, and the local wall buckling constant C from eq. T7 was taken as 0.2.

**L=3.5 meters.** - For this beam length the tip mass is  $0.01 \text{ lb} \cdot \text{sec}^2/\text{in}$  (3.86 lbs). The weights for the four deployable beams considered in this paper are shown in figure 1, and the associated beam diameters are shown in figure 2. The dashed lines indicate the results when beam mass is not included, showing no appreciable change in beam weight or diameter has taken place. In figure 1 the 3 mil aluminum beam is the lightest over the low frequency range, while the coilable longeron beam is lighter for frequencies greater than 0.2 Hz. For this relatively short length the three tubular beams are governed primarily by the frequency constraint, while the coilable longeron beam is governed by the root moment/stowage constraint at low frequencies and by root moment/frequency for the higher frequencies.

For this short beam length the beam weights are quite low for all four concepts, however, as seen in figure 2 there is quite a difference in beam diameters. For the three tubular beams the required diameter is 3 inches or less while the required diameter for the coilable longeron beam ranges from 3 to 8 inches. The beam diameter will greatly influence the stowage volume and deployment weight required for the deployable beams. In the present paper the only weight considered for the deployable beams is that required for structural performance. For all of the beams there will be a system weight required to accomplish deployment. For the STEM and the coilable longeron beams there is a mechanical deployment canister required that is typically several times the weight of the beam structure<sup>8</sup>. For the inflatable beams there is the weight associated with the pressurization system that must be considered. The details of these auxiliary weights are beyond the scope of the present paper, however, these weights will be a strong function of beam diameter.

**L=7 meters.** - For this beam length the tip mass is  $.04 \text{ lb} \cdot \text{sec}^2/\text{in}$  (15.44 lbs). These beam weights are shown in figure 3 while the corresponding beam diameters are shown in figure 4. Once again the dashed lines indicate results not including the mass of the beam. Some variation is seen in the beam weights and diameters, particularly in the frequency constraint for increasing frequency. The rigidizable beam is the lightest for very low frequencies followed by the 3 mil aluminum beam up to about .35 Hz, after which the coilable longeron beam is the lightest. The strength constrained weight of a tubular beam is inversely proportional to the assumed thickness  $t$ , when the contribution of  $m_{\text{beam}}$  is relatively small. Thus, if the aluminum thickness could be doubled to 6 mils, the weight of the aluminum beam in the low frequency range would be reduced by a factor of about two. The tubular beam diameters are seen from figure 4 to be about one half of the coilable longeron beam diameters. In fact the coilable longeron diameter reaches 20 inches, the size limit of practicality for these beams.

**L=14 meters.** - For this beam length the tip mass is  $.16 \text{ lb} \cdot \text{sec}^2/\text{in}$  (61.76 lbs). These beam weights are shown in figure 5 while the corresponding beam diameters are shown in figure 6. At this length a significant change can be seen between including the beam mass and not including the beam mass, for both the beam weights and diameters. This is especially apparent when the root moment constraint dominates and when the frequency increases. All beam weights increased except for the low frequency range of the STEM beam. The rigidizable beam has the lowest weight up to about 0.2 Hz, beyond which the 3 mil aluminum beam is the lightest. For these longer lengths it will probably be necessary to limit the design frequency to 0.4 Hz or less to keep the beam weights practical. The corresponding diameters for these beams are shown in figure 6. The diameters, in general, also increased except for the coilable longeron which stayed the same. The reason that the diameter of the coilable longeron stayed the same, is that in

solving equation C6 for R, the term  $\frac{A_l^2}{M}$ , when beam mass is included in the moment, is nearly equal to  $\frac{A_l^2}{M}$  when the beam mass is not included in the moment. The coilable longeron beam diameter is out of the practical design range for frequencies greater than 0.3 Hz while the aluminum beam is probably impractically large over the entire range. Even the rigidizable begins to assume impractical diameters for frequencies above 0.3 Hz. The addition of the beam mass does not change the conclusions based on beam diameters because the aluminum is even more impractical over the entire range than it was without the beam mass. Also the practical design limit for the other beams ends at about 0.3-0.4 Hz, up to which point the addition of the beam mass makes virtually no difference.

**L=28 meters.** - For this length the beam tip mass was not increased over that for the 14 m beam. If the tip mass were increased as a function of a square of the length this would result in a tip weight of about 240 lbs. This was not considered to be practical for most applications so the tip weight was kept at 61.76 lbs. It should also be pointed out that this 28 m length is the same as the IAE (Inflatable Antenna Experiment) support boom length<sup>4</sup>. In fact this length and tip mass are representative of design conditions for large inflatable reflector applications. The beam weights for this length are shown in figure 7 and the corresponding diameters are shown in figure 8. It is obvious at this point that the inclusion of the beam mass makes an increasingly greater difference as the beam is lengthened. The beam weights demonstrate the greatest difference between including the beam mass and not including the beam mass, in the cases of the coilable longeron and 3 mil aluminum beams. In figure 7 it can be seen that the rigidizable beam is lightest over most of the practical frequency range. Because of the rapid increase in beam weight and diameter with frequency, the design frequency for such structures would probably have to be restricted to 0.2 Hz or less. By restricting the design frequencies to 0.2 Hz or less, the effects of including the beam mass are not as significant as they could be for a higher frequency. Though, the effects are still significant where the root moment constraint dominates.

**Weight as a function of length** - The equations in the *Mathematica* program were modified to accept a constant frequency, and become functions of beam length instead (Appendix B). In figure 9 the weights of the four beam concepts are plotted as a function of length for a fixed natural frequency of 0.2 Hz. This figure demonstrates that the rigidizable material beam is quite efficient for the longer lengths. In figure 10 the same weight curves are presented with the addition of a 0.006" thick aluminum tubular beam. As can be seen from the figure this thickness results in an aluminum beam with the same efficiency as the rigidizable material beam. Because of the relatively simple deployment process for the aluminum beam, a research effort should be conducted to determine if such a beam could be developed. Figure 11 shows the effect on the moment constraint of increasing  $\alpha$  to 2. Both aluminum beams and the coilable longeron become less efficient and increase in weight, especially for increasing length. The STEM and rigidizable beams appear to remain nearly the same.

## CONCLUDING REMARKS

The purpose of this investigation was to develop and demonstrate a design methodology for tubular, rigidizable, space beam structures and improve on the accuracy from that of reference 1, by including the mass of the beam. This methodology was applied to a new class of rigidizable beams to permit a rational comparison with alternate deployable

concepts. Specifically the rigidizable beams were compared with the STEM and coilable longeron beams on a weight and diameter basis.

A series of equations that included the mass of the beam and a tip mass were developed for the weight and diameter for each of the concepts, for the condition of a long beam cantilevered from a spacecraft. The two design requirements considered were a lowest natural frequency constraint and a root moment constraint imposed by a lateral acceleration loading. Although it is difficult to draw completely general conclusions as to the relative efficiency of the different beam concepts, representative small spacecraft operational conditions were assumed to enable a comparison to be made.

The two primary rigidizable concepts investigated were a 0.011" thick KEVLAR fabric impregnated with a rigidizable matrix and a 0.003" thick aluminum tube which is rigidized by pressure yielding the material. Beam lengths ranging from 3.5 m to 28 m were investigated for a frequency range from 0.02 Hz to 1 Hz. The strength constraint imposed was that the beam be required to withstand a 0.015 g lateral loading. The tip mass was inertially similar to the mass of a distributed solar array. Results from this study are as follows:

- 1) Because of the discrete practical thickness constraints imposed on the different tubular concepts the active design constraint, frequency or strength, is a function of beam length, material properties, and tip mass. For the shorter lengths and higher frequency requirements the frequency constraint is active, while for longer lengths and lower frequency requirements the strength constraint is active.
- 2) The three tubular beams investigated, the KEVLAR rigidizable, the aluminum, and the steel STEM tend to have smaller diameters than the coilable longeron beam.
- 3) For shorter length applications the 3 mil aluminum beam is the lightest and most compact for low natural frequency requirements.
- 4) For longer length applications the KEVLAR rigidizable beam becomes attractive from a weight and diameter point of view.
- 5) If thicker (~ 0.006") aluminum beam concepts could be developed, they would be very efficient over the entire range of parameters investigated. Since the rigidizable aluminum beam is so conceptually simple, it is recommended that alternate wall constructions be investigated to extend its range of application.
- 6) The addition of the beam mass results in increased weight for the root moment constraints, and increased weight as frequency increases, for the frequency constraints. These effects become increasingly apparent as the beam is lengthened.
- 7) The design methodology developed herein permits a rational assessment of the effect of spacecraft requirements (frequency and strength) on deployable beam weight and diameter. It is recommended that this methodology be used early in the design process to assist in establishing rational and reasonable spacecraft design requirements. Conducting a thorough sensitivity study of deployable beam weight and diameter to spacecraft design requirements early in the design process should lead to the most robust design at the lowest cost in terms of weight and stowage efficiency.

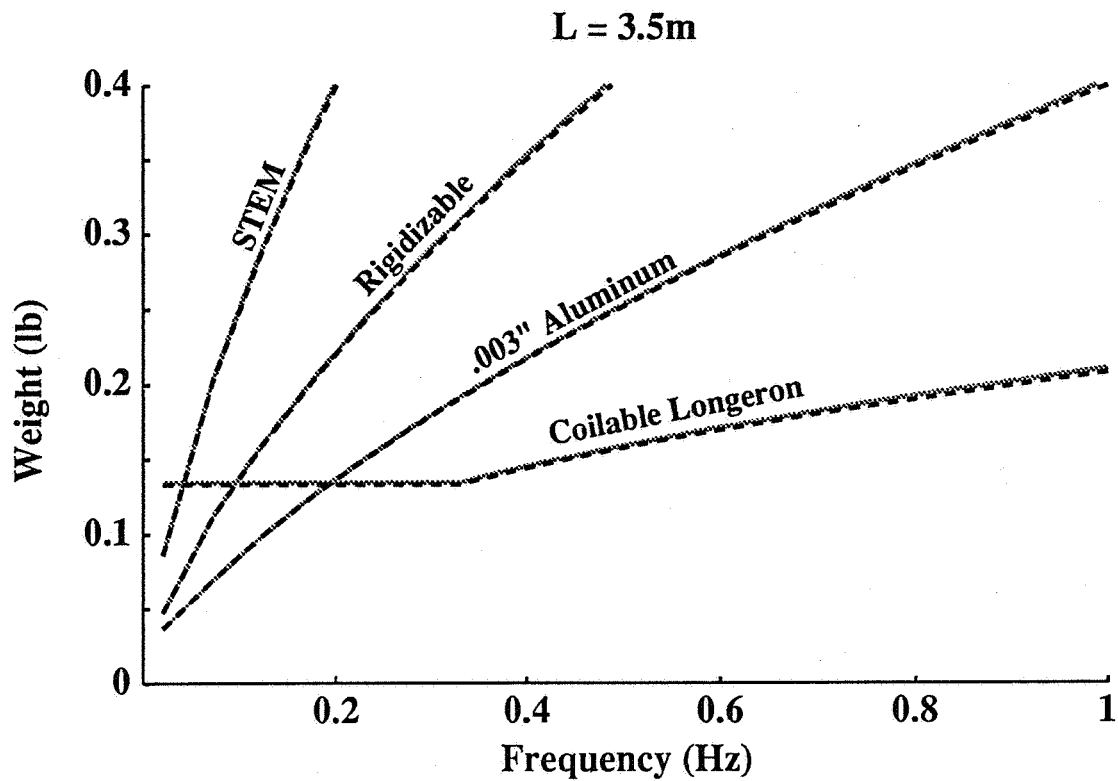


Figure 1.- Deployable beam weight as a function of frequency for  $L = 3.5\text{m}$ .  
(Dashed lines indicate weight without beam mass.)

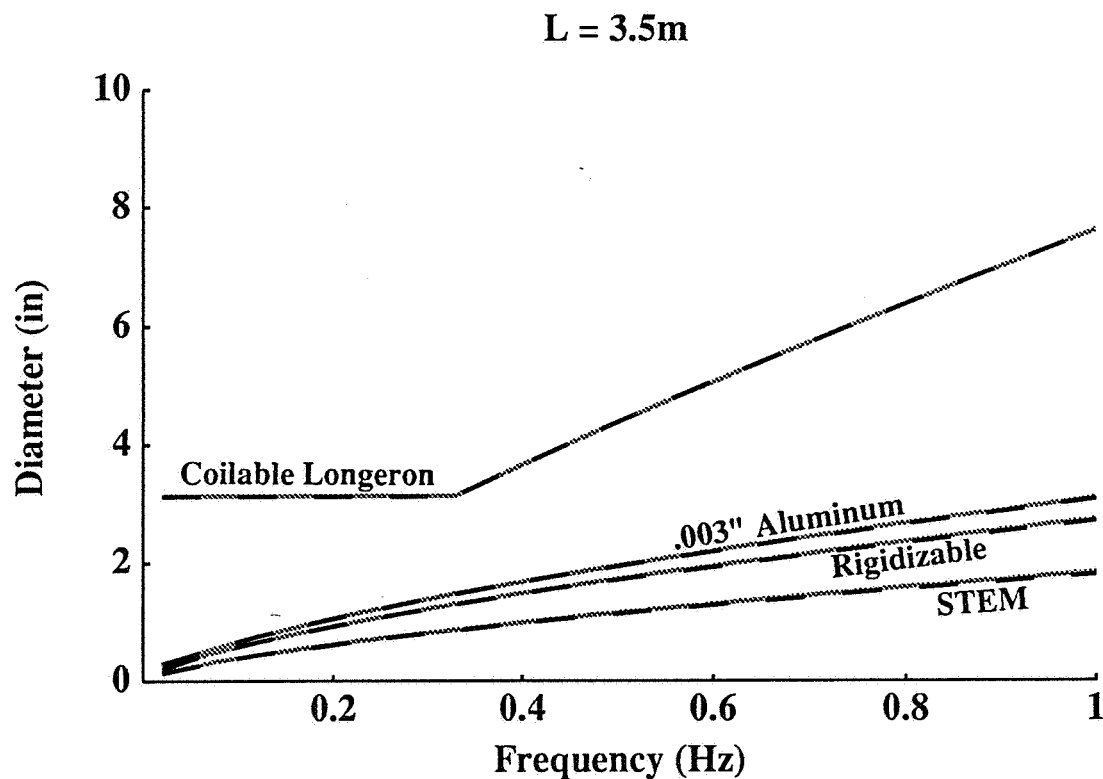


Figure 2.- Deployable beam diameter as a function of frequency for  $L = 3.5\text{m}$ .  
(Dashed lines indicate diameter without beam mass.)

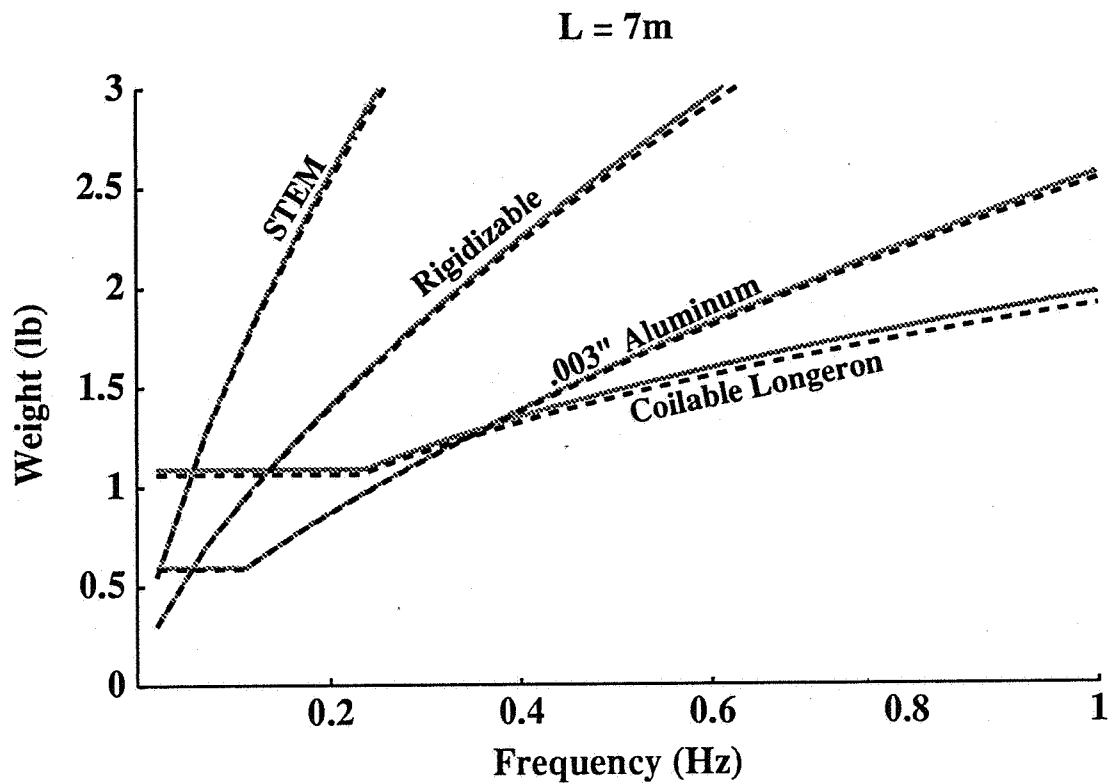


Figure 3.- Deployable beam weight as a function of frequency for  $L = 7m$ .  
(Dashed lines indicate weight without beam mass.)

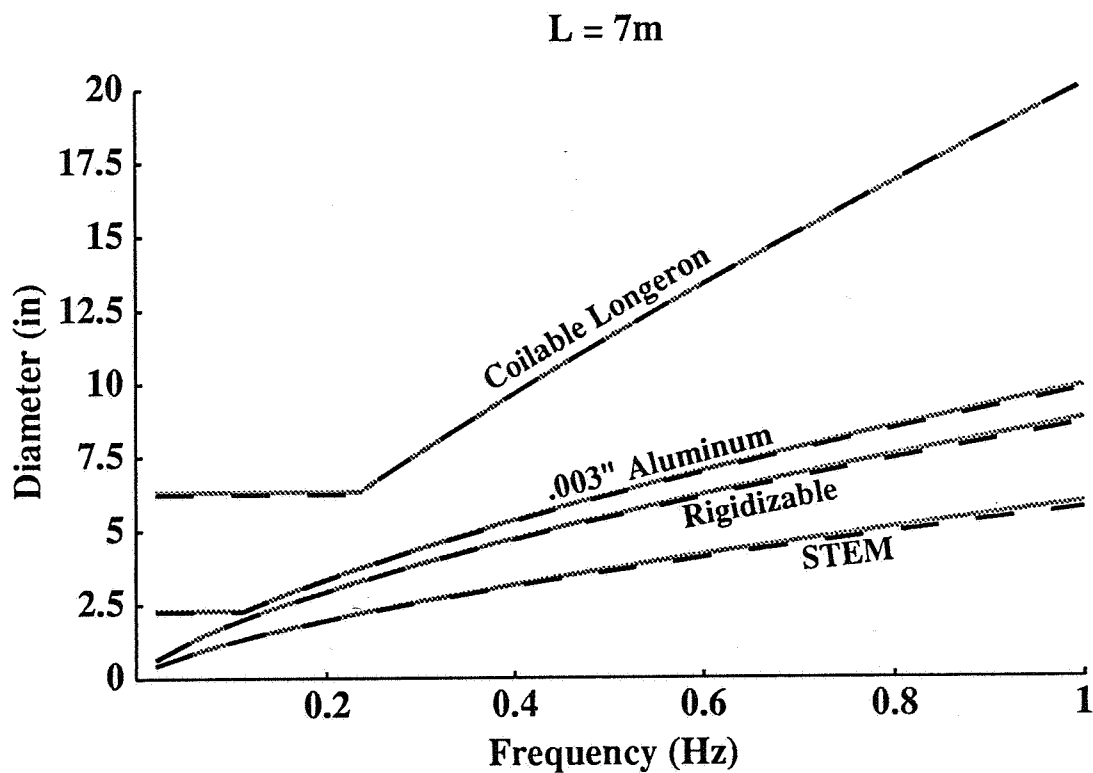


Figure 4.- Deployable beam diameter as a function of frequency for  $L = 7m$ .  
(Dashed lines indicate diameter without beam mass.)

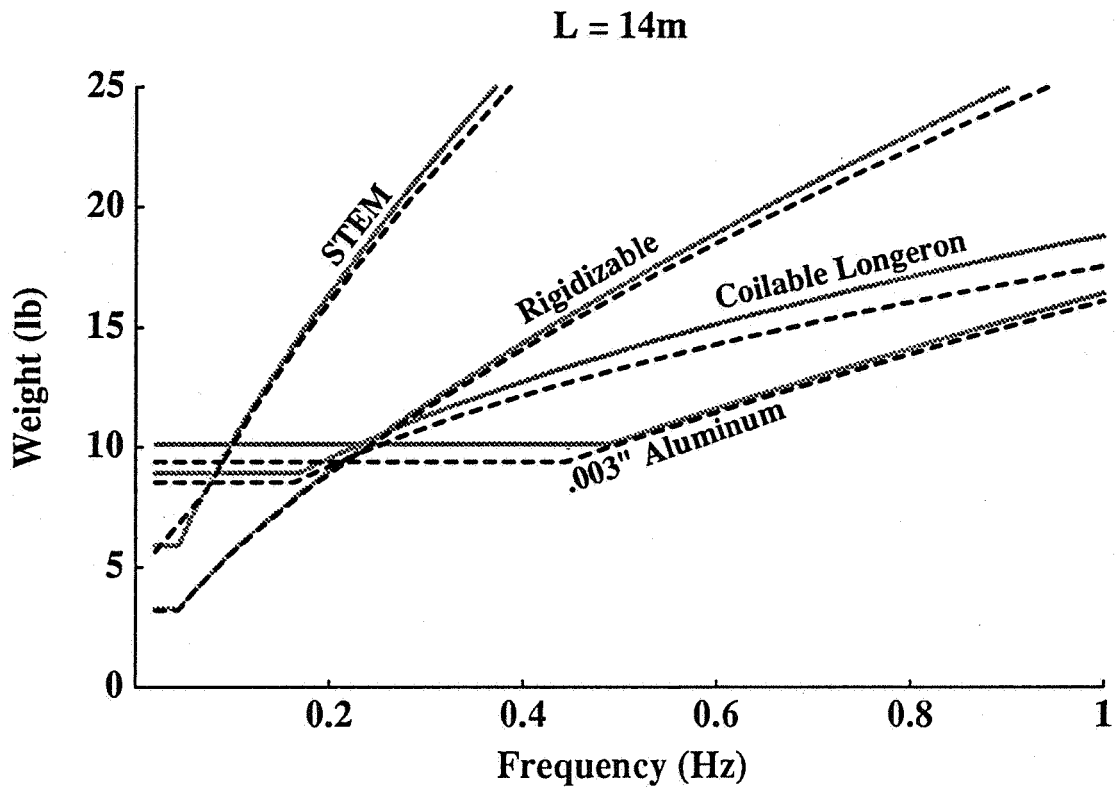


Figure 5.- Deployable beam weight as a function of frequency for  $L = 14m$ .  
(Dashed lines indicate weight without beam mass.)

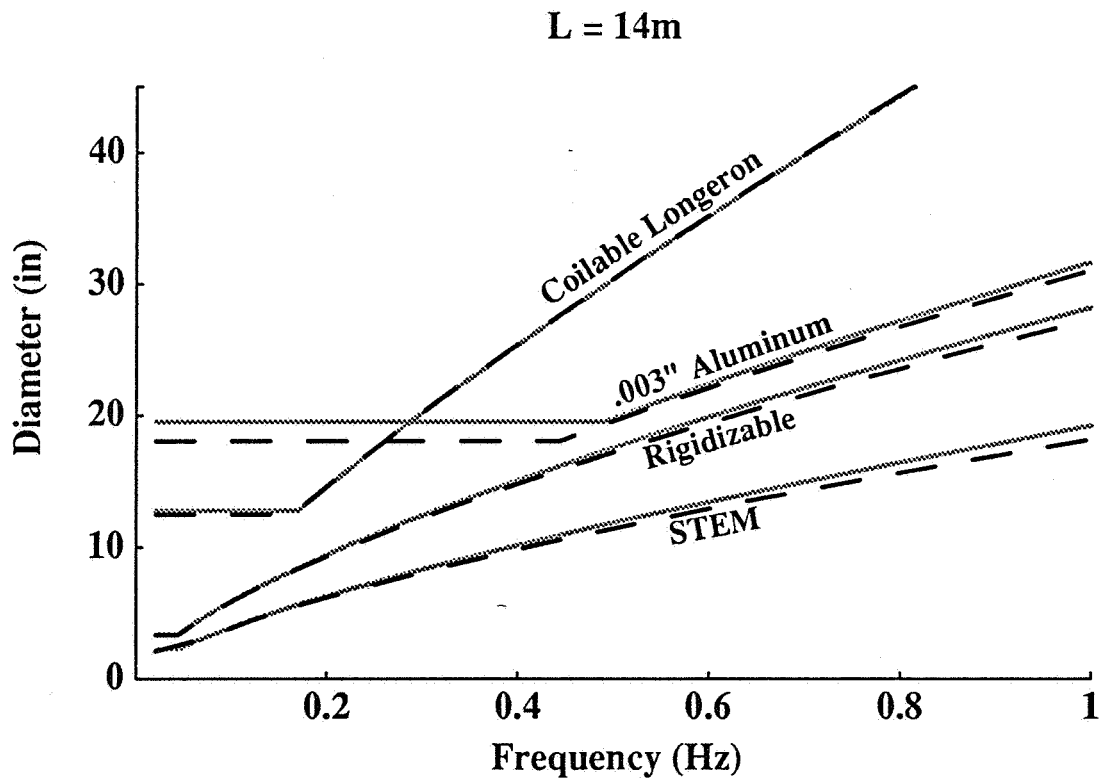


Figure 6.- Deployable beam diameter as a function of frequency for  $L = 14m$ .  
(Dashed lines indicate diameter without beam mass.)



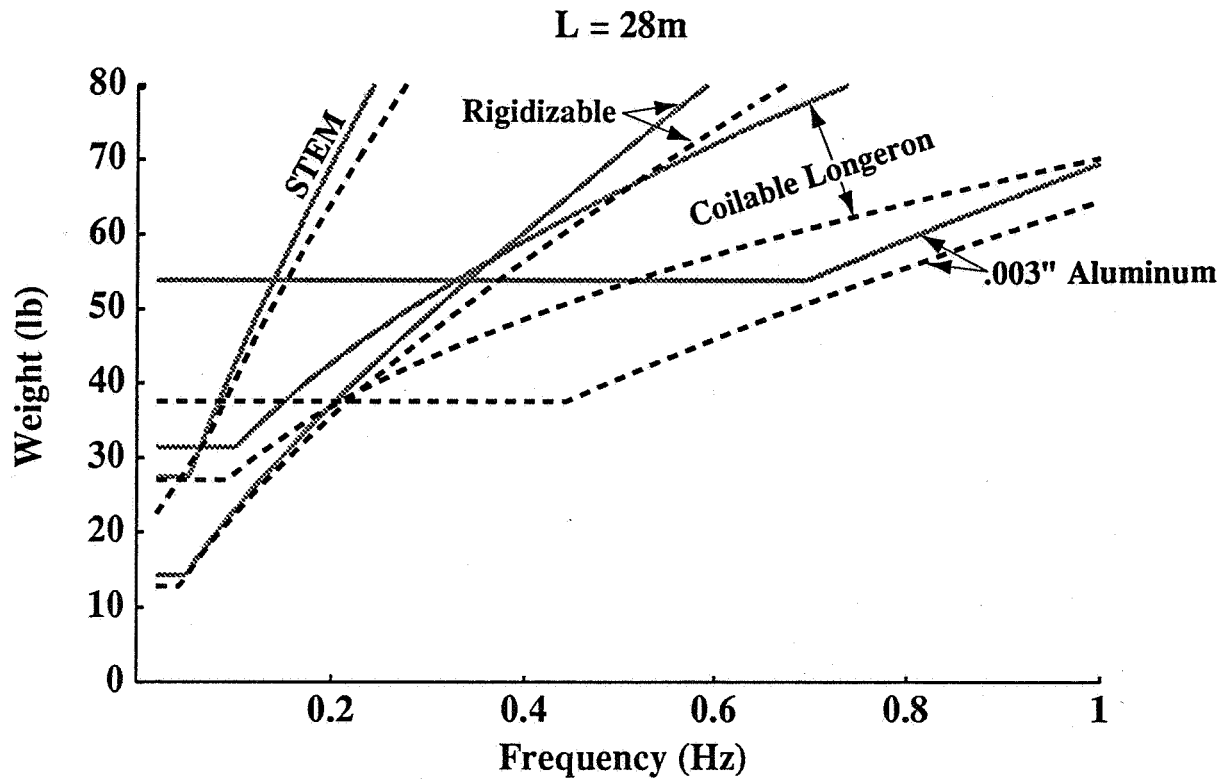


Figure 7.- Deployable beam weight as a function of frequency for  $L = 28m$ .  
(Dashed lines indicate weight without beam mass.)

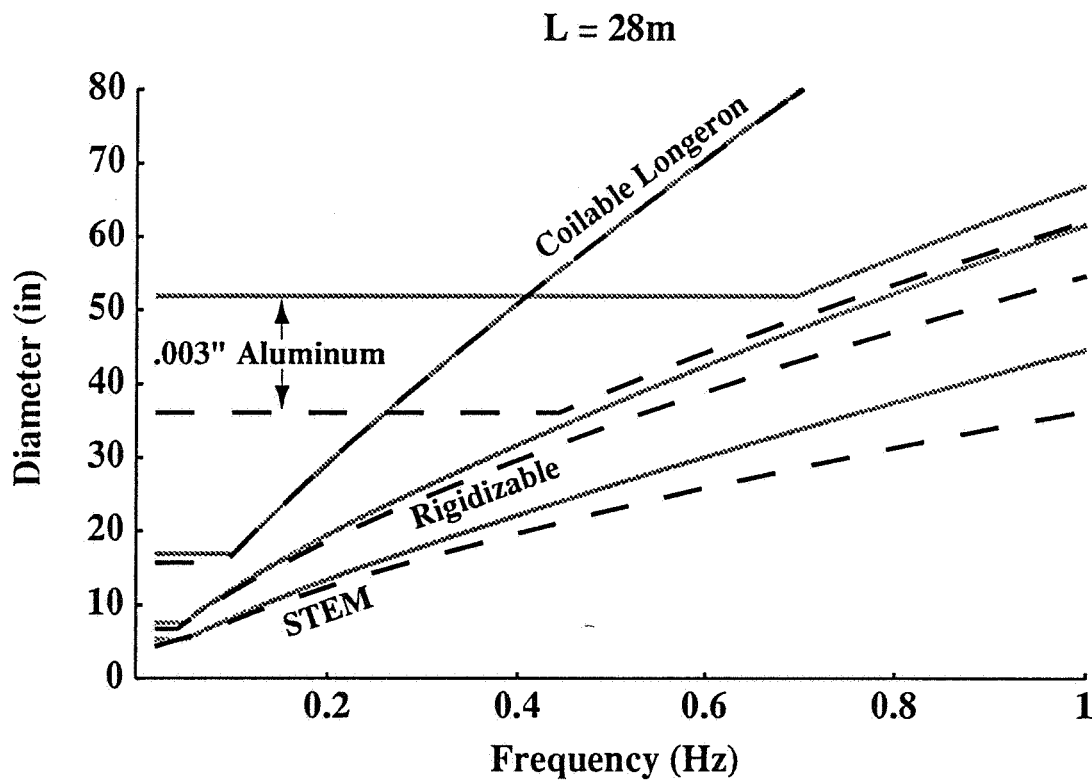


Figure 8.- Deployable beam diameter as a function of frequency for  $L = 28m$ .  
(Dashed lines indicate diameter without beam mass.)

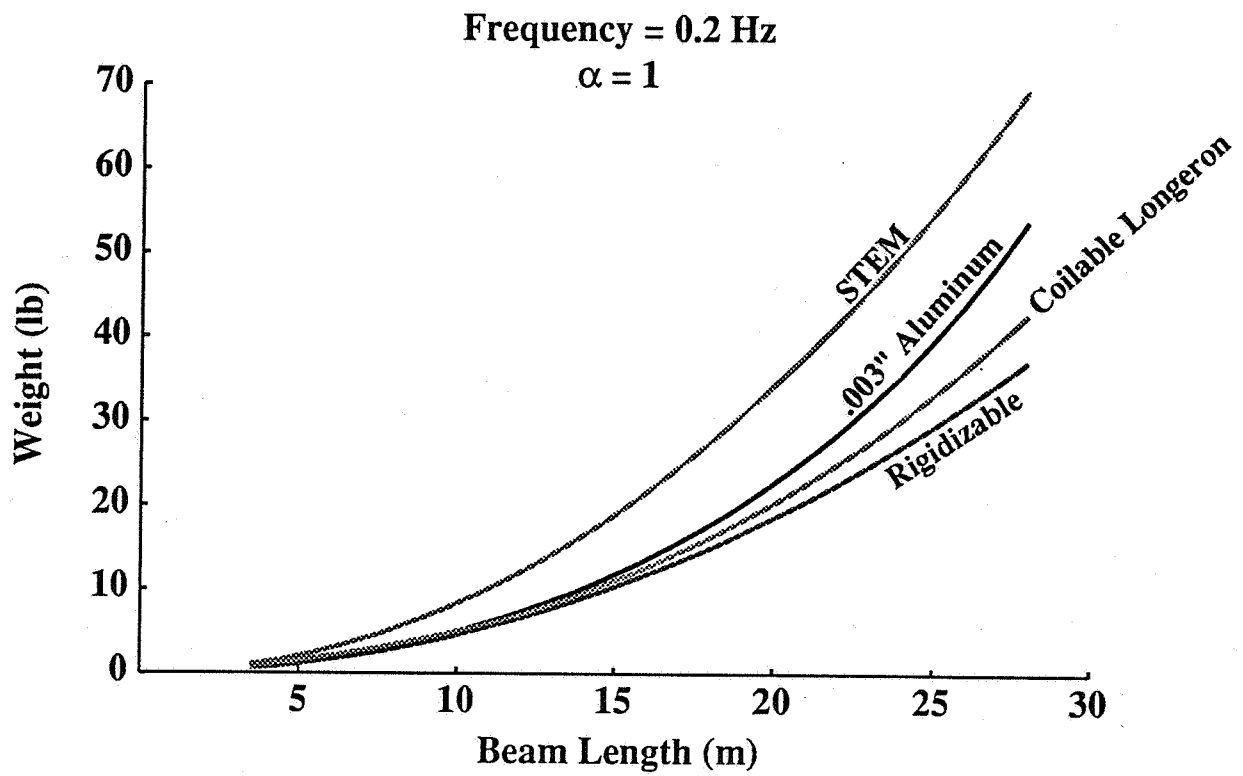


Figure 9.- Beam weight as a function of length for  $f = 0.2$  Hz.

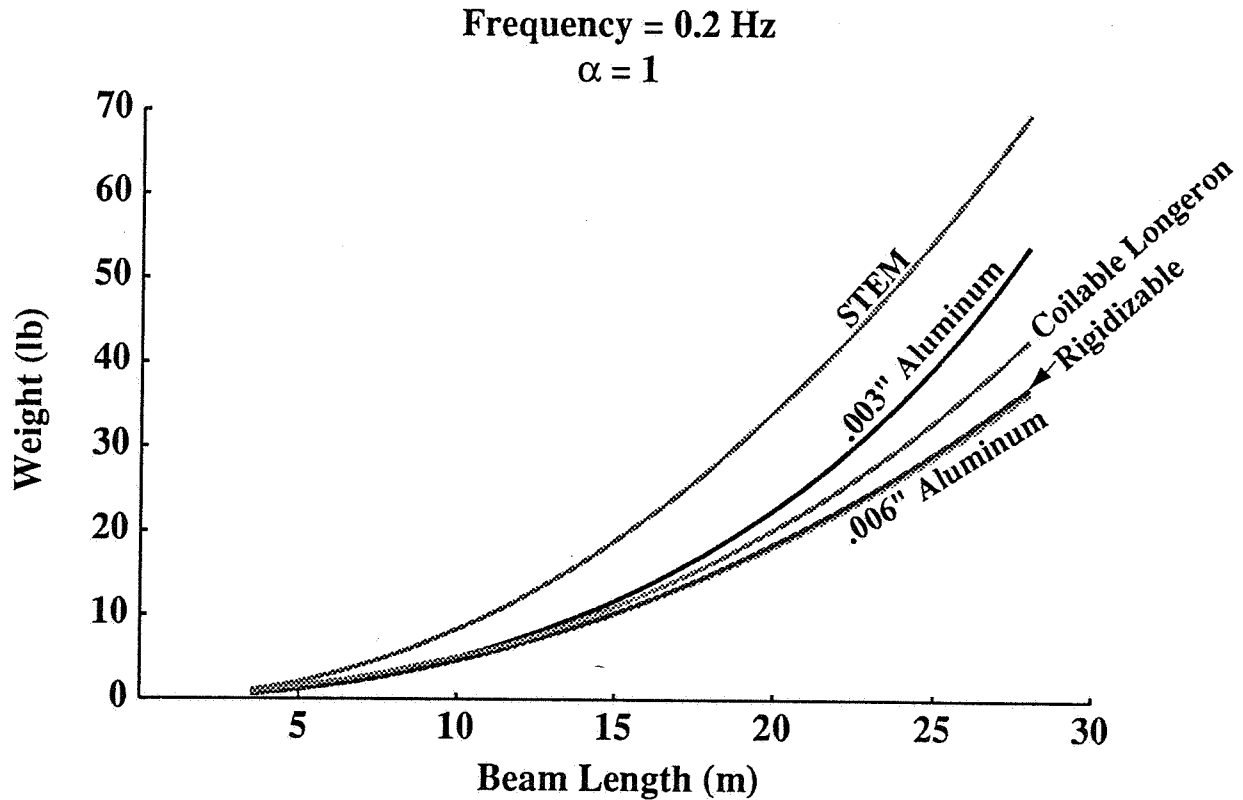


Figure 10.- Beam weight as a function of length for  $f = 0.2$  Hz with additional aluminum curve for .006" thickness wall.

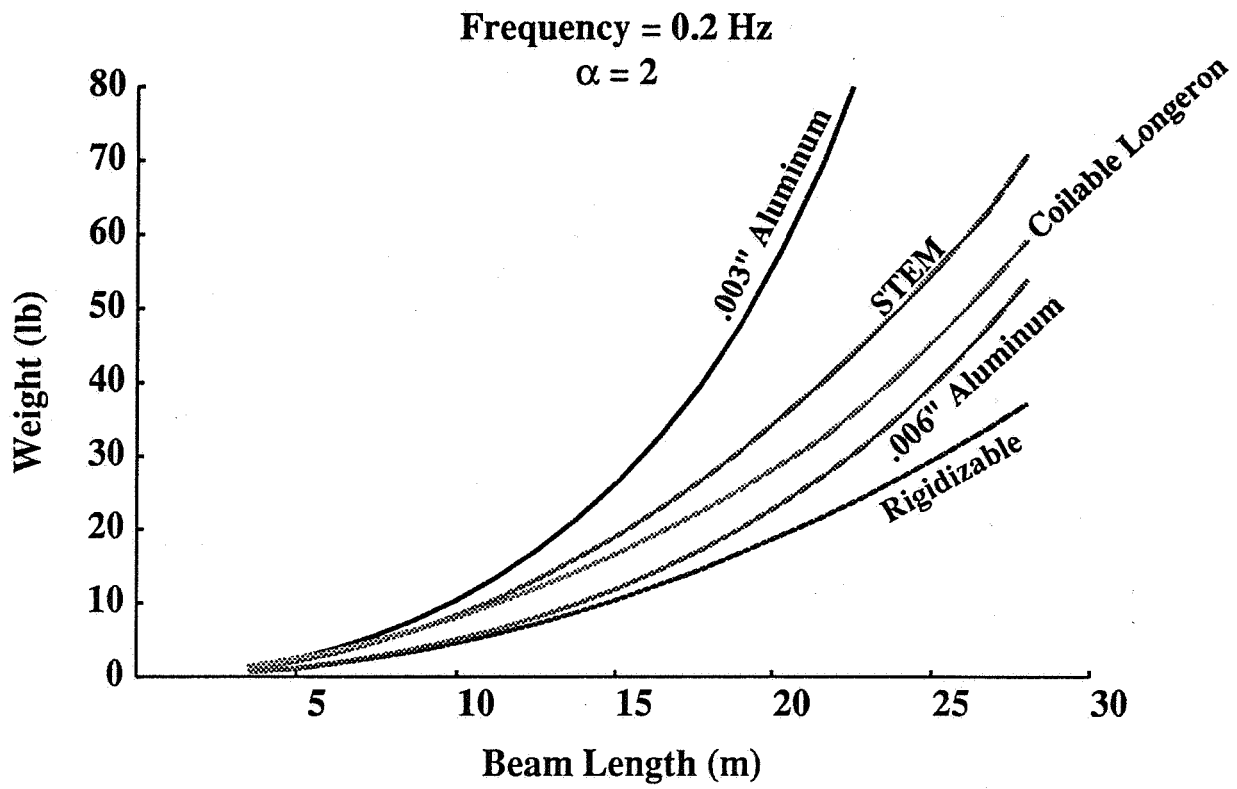


Figure 11.- Beam weight as a function of length for  $f = 0.2$  Hz with additional aluminum curve for .006" thickness wall.

Off[General::spell1]; Off[General::spell];

(\*Variable Definitions

alpha = dynamic overshoot factor  
c = local wall buckling constant  
mtip = mass applied to the tip of the beam (lb sec<sup>2</sup> / in)  
len = length of the beam (in)  
e = modulus of elasticity (psi)  
rho = weight density of beam (lb/in<sup>3</sup>)  
t = wall thickness of beam (in)  
g = gravitational acceleration (in / sec<sup>2</sup>)  
r = radius of beam (in)  
a = area of longeron (in<sup>2</sup>)  
epsilon = longeron allowable strain (in/in)  
fs = factor of safety for strength (root moment) constraint  
sg = lateral acceleration loading coefficient  
moment = root moment (in lb)  
momn = root moment neglecting the beam mass (in lb)  
i = moment of inertia (in<sup>4</sup>)  
f = natural bending frequency (Hz)  
mbeam = mass of beam (lb sec<sup>2</sup> / in)  
wmom = weight of tubular beam with root moment constraint (lb)  
wfreq = weight of tubular beam with frequency constraint (lb)  
dmom = diameter of tubular beam with root moment constraint (in)  
dfreq = diameter of tubular beam with frequency constraint (in)  
wmomn = weight of tubular beam with root moment constraint,  
neglecting the mass of the beam (lb)  
wfreqn = weight of tubular beam with frequency constraint,  
neglecting the mass of the beam (lb)  
dmomn = diameter of tubular beam with root moment constraint,  
neglecting the mass of the beam (in)  
dfreqn = diameter of tubular beam with frequency constraint,  
neglecting the mass of the beam (in)  
wcl = weight of coilable longeron beam with frequency and root  
moment constraints (lb)  
wcls = weight of coilable longeron beam with root moment and  
allowable strain constraints (lb)  
dcl = diameter of coilable longeron beam with frequency and root  
moment constraints (in)  
dcls = diameter of coilable longeron beam with root moment and  
allowable strain constraints (in)  
wcln = weight of coilable longeron beam with frequency and root  
moment constraints, neglecting the mass of the beam (lb)  
wcls n = weight of coilable longeron beam with root moment and  
allowable strain constraints , neglecting the mass of the beam (lb)  
dcln = diameter of coilable longeron beam with frequency and root  
moment constraints, neglecting the mass of the beam (in)  
dcls n = diameter of coilable longeron beam with root moment and  
allowable strain constraints, neglecting the mass of the beam (in)

\*)

```
Clear[alpha, c, mtip, len, e, rho, t, g, r, fs, sg,
moment, momn, i, f, mbeam, wmom, wfreq, dmom,
dfreq, wmomn, wfreqn, dmomn, dfreqn];
```

*(\*Global Variable Assignments\*)*

```
alpha=1; c=0.2; mtip=.16; len=1400/2.54; g=386; fs=1; sg=.015;
momn=alpha sg g mtip len;
```

*(\*3 mil Aluminum\*)*

```
e=10*10^6; rho=.1; t=.003; i=Pi r^3 t;
mbeam=rho 2 Pi r t len/g;
moment=alpha sg g (mtip len + mbeam len/2);
```

```
dfreq[f_]:= 2 (r/. FindRoot[1/2/Pi Sqrt[3*
e i /(len^3 (mtip +.227 mbeam))]==f,{r,1}]);
```

```
dmom= 2 (r/. Solve[fs moment / (Pi c e t^2)==r,{r}]);
```

```
dfreqn[f_]:= 2 (r/. FindRoot[1/2/Pi Sqrt[3*
e i /(len^3 mtip)]==f,{r,1}]);
```

```
dmomn= 2 (r/. Solve[fs momn / (Pi c e t^2)==r,{r}]);
```

```
wfreq[f_]:= rho Pi t len dfreq[f];
```

```
wmom= rho Pi t len dmom;
```

```
wfreqn[f_]:= rho Pi t len dfreqn[f];
```

```
wmomn= rho Pi t len dmomn;
```

```
p1=Plot[Max[wmom,wfreq[f]],{f,.02,1},
PlotStyle->{GrayLevel[.6]},
DisplayFunction->Identity];
```

```
p5=Plot[Max[dmom,dfreq[f]],{f,.02,1},
PlotStyle->{GrayLevel[.6]},
DisplayFunction->Identity];
```

```
p9=Plot[Max[wmomn,wfreqn[f]],{f,.02,1},
PlotStyle->{Dashing[{.01}]},
DisplayFunction->Identity];
```

```
p13=Plot[Max[dmomn,dfreqn[f]],{f,.02,1},
PlotStyle->{Dashing[{.04}]},
DisplayFunction->Identity];
```

(\*STEM\*)

Clear[e, rho, t, r, i, moment, mbeam, wfreq, wmom,  
wfreqn, wmomn, dfreq, dmom, dfreqn, dmomn];

e=30\*10^6; rho=.3; t=.005; i=Pi r^3 t;  
mbeam=rho 2 Pi r t len/g;  
moment=alpha sg g (mtip len + mbeam len/2);

dfreq[f\_]:= 2 (r/. FindRoot[1/2/Pi Sqrt[3\*  
e i /(len^3 (mtip +.227 mbeam))]==f,{r,1}]);

dmom= 2 (r/. Solve[fs moment / (Pi c e t^2)==r,{r}]);

dfreqn[f\_]:= 2 (r/. FindRoot[1/2/Pi Sqrt[3\*  
e i /(len^3 mtip)]==f,{r,1}]);

dmomn= 2 (r/. Solve[fs momn / (Pi c e t^2)==r,{r}]);

wfreq[f\_]:= rho Pi t len dfreq[f];

wmom= rho Pi t len dmom;

wfreqn[f\_]:= rho Pi t len dfreqn[f];

wmomn= rho Pi t len dmomn;

p2=Plot[Max[wmom,wfreq[f]],{f,.02,1},  
PlotStyle->{GrayLevel[.6]},  
DisplayFunction->Identity];

p6=Plot[Max[dmom,dfreq[f]],{f,.02,1},  
PlotStyle->{GrayLevel[.6]},  
DisplayFunction->Identity];

p10=Plot[Max[wmomn,wfreqn[f]],{f,.02,1},  
PlotStyle->{Dashing[{.01}]},  
DisplayFunction->Identity];

p14=Plot[Max[dmomn,dfreqn[f]],{f,.02,1},  
PlotStyle->{Dashing[{.04}]},  
DisplayFunction->Identity];

(\*Rigidizable Beam\*)

Clear[e, rho, t, r, i, moment, mbeam, wfreq, wmom,  
wfreqn, wmomn, dfreq, dmom, dfreqn, dmomn];

e=4\*10^6; rho=.05; t=.011; i=Pi r^3 t;  
mbeam=rho 2 Pi r t len/g;  
moment=alpha sg g (mtip len + mbeam len/2);

dfreq[f\_]:= 2 (r/. FindRoot[1/2/Pi Sqrt[3\*  
e i /(len^3 (mtip +.227 mbeam))]==f,{r,1}]);

dmom= 2 (r/. Solve[fs m / (Pi c e t^2)==r,{r}]);

dfreqn[f\_]:= 2 (r/. FindRoot[1/2/Pi Sqrt[3\*  
e i /(len^3 mtip)]==f,{r,1}]);

dmomn= 2 (r/. Solve[fs momn /(Pi c e t^2)==r,{r}]);

wfreq[f\_]:= rho Pi t len dfreq[f];

wmom= rho Pi t len dmom;

wfreqn[f\_]:= rho Pi t len dfreqn[f];

wmomn= rho Pi t len dmomn;

p3=Plot[Max[wmom,wfreq[f]],{f,.02,1},  
PlotStyle->{GrayLevel[.6]},  
DisplayFunction->Identity];

p7=Plot[Max[dmom,dfreq[f]],{f,.02,1},  
PlotStyle->{GrayLevel[.6]},  
DisplayFunction->Identity];

p11=Plot[Max[wmomn,wfreqn[f]],{f,.02,1},  
PlotStyle->{Dashing[{.01}]},  
DisplayFunction->Identity];

p15=Plot[Max[dmomn,dfreqn[f]],{f,.02,1},  
PlotStyle->{Dashing[{.04}]},  
DisplayFunction->Identity];

(\*Coilable Longeron Beam\*)

Clear[e, rho, a, r, i, moment, mbeam, wcl, wcls, epsilon,  
dcl, dcls, wcln, wclsn, dcln, dclsn];

epsilon=.0133; rho=.07; e=7.5\*10^6; i=1.5 a r^2;  
mbeam=3.4 (3 rho a len) / g;  
moment=alpha sg g (mtip len + mbeam len/2);

dcl[f\_]:= 2 (r/. FindRoot[{1.5 Pi e a^2 /(4 (1.14)^2  
\*fs moment)==r, 1/2/Pi Sqrt[3 e i /(len^3 (mtip +.227  
\*mbeam))]==f},{a,.1},{r,3}][[2]]);

dcls= 2 (r/. FindRoot[{1.5 Pi e a^2 /(4 (1.14)^2  
\*fs moment)==r, Sqrt[4 a/Pi]/(2 r)==epsilon},{a,.1},{r,3}][[2]]);

dcln[f\_]:= 2 (r/. FindRoot[{1.5 Pi e a^2 /(4 (1.14)^2  
\*fs momn)==r, 1/2/Pi Sqrt[3 e i /(len^3 mtip)]==f},{a,.1},{r,3}][[2]]);

dclsn= 2 (r/. FindRoot[{1.5 Pi e a^2 /(4 (1.14)^2  
\*fs momn)==r, Sqrt[4 a/Pi]/(2 r)==epsilon},{a,.1},{r,3}][[2]]);

wcl[f\_]:= 3.4 3 rho len (a/. FindRoot[{1.5 Pi e a^2 /(4 (1.14)^2  
\*fs moment)==r, 1/2/Pi Sqrt[3 e i /(len^3 (mtip +.227  
\*mbeam))]==f},{a,.1},{r,3}][[1]]);

wcls= 3.4 3 rho len (a/. FindRoot[{1.5 Pi e a^2 /(4 (1.14)^2  
\*fs moment)==r, Sqrt[4 a/Pi]/(2 r)==epsilon},{a,.1},{r,3}][[1]]);

wcln[f\_]:= 3.4 3 rho len (a/. FindRoot[{1.5 Pi e a^2 /(4 (1.14)^2  
\*fs momn)==r, 1/2/Pi Sqrt[3 e i /(len^3 mtip)]==f},{a,.1},{r,3}][[1]]);

wclsn= 3.4 3 rho len (a/. FindRoot[{1.5 Pi e a^2 /(4 (1.14)^2  
\*fs momn)==r, Sqrt[4 a/Pi]/(2 r)==epsilon},{a,.1},{r,3}][[1]]);

p4=Plot[Max[wcls, wcl[f]],{f,.02,1},  
PlotStyle->{GrayLevel[.6]},  
DisplayFunction->Identity];

p8=Plot[Max[dcls, dcl[f]],{f,.02,1},  
PlotStyle->{GrayLevel[.6]},  
DisplayFunction->Identity];

p12=Plot[Max[wclsn, wcln[f]],{f,.02,1},  
PlotStyle->{Dashing[{.01]}},  
DisplayFunction->Identity];

p16=Plot[Max[dclsn, dcln[f]],{f,.02,1},  
PlotStyle->{Dashing[{.04]}},  
DisplayFunction->Identity];



*(\*p1,p2,p3,&p4 plot the weight equations including the mass of the beam.*

*p9,p10,p11,&p12 plot the weight equations not including the mass of the beam, using dashed lines for comparison.*

*Similarly for p5-p8, and p13-p16, except they plot the diameters.\*)*

```
Show[p1,p2,p3,p4,p9,p10,p11,p12,  
  DisplayFunction->$DisplayFunction,  
  AxesLabel->{"Frequency (Hz)","Weight (lb)"},  
  PlotRange->{{0,1},{0,25}},  
  PlotLabel->"Alum.,STEM,Rigid.,& C.L. for 14m"];
```

```
Show[p5,p6,p7,p8,p13,p14,p15,p16,  
  DisplayFunction->$DisplayFunction,  
  AxesLabel->{"Frequency (Hz)","Diameter (in)"},  
  PlotRange->{{0,1},{0,45}},  
  PlotLabel->"Alum.,STEM,Rigid.,& C.L. for 14m"];
```

**Off[General::spell1]; Off[General::spell];**

**(\*Variable Definitions**

alpha = dynamic overshoot factor  
c = local wall buckling constant  
mtip = mass applied to the tip of the beam (lb sec<sup>2</sup> / in)  
len = length of the beam (m) NOTE: The beam length in this program is specified  
in meters for plotting purposes.  
e = modulus of elasticity (psi)  
rho = weight density of beam (lb/in<sup>3</sup>)  
t = wall thickness of beam (in)  
g = gravitational acceleration (in / sec<sup>2</sup>)  
r = radius of beam (in)  
a = area of longeron (in<sup>2</sup>)  
epsilon = longeron allowable strain (in/in)  
fs = factor of safety for strength (root moment) constraint  
sg = lateral acceleration loading coefficient  
moment = root moment (in lb)  
i = moment of inertia (in<sup>4</sup>)  
f = natural bending frequency (Hz)  
mbeam = mass of beam (lb sec<sup>2</sup> / in)  
wmom = weight of tubular beam with root moment constraint (lb)  
wfreq = weight of tubular beam with frequency constraint (lb)  
wcl = weight of coilable longeron with frequency and root  
moment constraints (lb)  
wcls = weight of coilable longeron beam with root moment and  
allowable strain constraints (lb)

**\*)**

**Clear[alpha, c, mtip, len, e, rho, t, g, r, fs, sg,  
moment, i, f, mbeam, wmom, wfreq];**

**(\*Global Variable Assignments\*)**

**alpha=1; c=0.2; mtip=.16; f=.2; g=386; fs=1; sg=.015;**

(\*3 mil Aluminum\*)

```
e=10*10^6; rho=.1; t=.003; i=Pi r^3 t;
mbeam=rho 2 Pi r t len 100/2.54/g;
moment=alpha sg g (mtip len 100/2.54 + mbeam len 100/2.54/2);

wfreq[len_]:= rho 2 Pi t len 100/2.54 (r/. FindRoot[1/2/Pi Sqrt[3*
e i /((len 100/2.54)^3 (mtip +.227 mbeam))]]
==f,{r,1}]);

wmom[len_]:= rho 2 Pi t len 100/2.54 (r/. Solve[fs moment / (Pi c e t^2)
==r,{r}]);

p1=Plot[Max[wmom[len],wfreq[len]],{len,3.5,28},
PlotStyle->{Hue[2/3]},
DisplayFunction->Identity];
```

(\*6mil Aluminum\*)

```
Clear[e, rho, t, r, i, moment, mbeam, wfreq, wmom];

e=10*10^6; rho=.1; t=.006; i=Pi r^3 t;
mbeam=rho 2 Pi r t len 100/2.54/g;
moment=alpha sg g (mtip len 100/2.54 + mbeam len 100/2.54/2);

wfreq[len_]:= rho 2 Pi t len 100/2.54 (r/. FindRoot[1/2/Pi Sqrt[3*
e i /((len 100/2.54)^3 (mtip +.227 mbeam))]]
==f,{r,1}]);

wmom[len_]:= rho 2 Pi t len 100/2.54 (r/. Solve[fs moment / (Pi c e t^2)
==r,{r}]);

p2=Plot[Max[wmom[len],wfreq[len]],{len,3.5,28},
PlotStyle->{Hue[.08]},
DisplayFunction->Identity];
```

*(\*STEM\*)*

```
Clear[e, rho, t, r, i, moment, mbeam, wfreq, wmom];

e=30*10^6; rho=.3; t=.005; i=Pi r^3 t;
mbeam=rho 2 Pi r t len 100/2.54/g;
moment=alpha sg g (mtip len 100/2.54 + mbeam len 100/2.54/2);

wfreq[len_]:= rho 2 Pi t len 100/2.54 (r/. FindRoot[1/2/Pi Sqrt[3*
e i /((len 100/2.54)^3 (mtip +.227 mbeam))])
==f,{r,1});

wmom[len_]:= rho 2 Pi t len 100/2.54 (r/. Solve[fs moment / (Pi c e t^2)
==r,{r}]);

p3=Plot[Max[wmom[len],wfreq[len]],{len,3.5,28},
PlotStyle->{Hue[1/3]},
DisplayFunction->Identity];
```

*(\*Rigidizable Beam\*)*

```
Clear[e, rho, t, r, i, moment, mbeam, wfreq, wmom];

e=4*10^6; rho=.05; t=.011; i=Pi r^3 t;
mbeam=rho 2 Pi r t len 100/2.54/g;
moment=alpha sg g (mtip len 100/2.54 + mbeam len 100/2.54/2);

wfreq[len_]:= rho 2 Pi t len 100/2.54 (r/. FindRoot[1/2/Pi Sqrt[3*
e i /((len 100/2.54)^3 (mtip +.227 mbeam))])
==f,{r,1});

wmom[len_]:= rho 2 Pi t len 100/2.54 (r/. Solve[fs moment / (Pi c e t^2)
==r,{r}]);

p4=Plot[Max[wmom[len],wfreq[len]],{len,3.5,28},
PlotStyle->{Hue[1]},
DisplayFunction->Identity];
```

*(\*Coilable Longeron Beam\*)*

Clear[e, rho, a, r, i, moment, mbeam, wcl, wcls, epsilon];

epsilon=.0133; rho=.07; e=7.5\*10^6; i=1.5 a r^2;

mbeam=3.4 (3 rho a len 100/2.54) / g;

moment=alpha sg g (mtip len 100/2.54 + mbeam len 100/2.54/2);

wcl[len\_]:= 3.4 3 rho len 100/2.54 (a/. FindRoot[{1.5 Pi e a^2/(4\*  
(1.14)^2 fs moment)==r, 1/2/Pi Sqrt[3 e i/((len 100/2.54)^3\*  
(mtip +.227 mbeam))]==f},{a,.1},{r,3}]);

wcls[len\_]:= 3.4 3 rho len 100/2.54 (a/. FindRoot[{1.5 Pi e a^2/(4\*  
(1.14)^2 fs moment)==r, Sqrt[4 a/Pi]/(2 r)==epsilon},{a,.2},  
{r,3}]);

p5=Plot[Max[wcls[len], wcl[len]],{len,3.5,28},  
PlotStyle->{Hue[1/2]},  
DisplayFunction->Identity];

Show[p1,p2,p3,p4,p5,  
DisplayFunction->\$DisplayFunction,  
AxesLabel->{"Beam Length (m)","Weight (lb)"},  
PlotRange->{{0,30},{0,70}},  
PlotLabel->"Frequency = 0.2 Hz"];

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